Bilateral direct comparison of Josephson Array Voltage Standards of the PTB (Germany) and PEL (Croatia)

Draft B report for EUROMET.EM-S28

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Abstract. A comparison of the 10 V Josephson array voltage standard of the Physikalisch-Technische Bundesanstalt (PTB), Germany, was made with that of the Primarni elektromagnetski laboratorij (PEL), Croatia, in April 2007. The results are in very good agreement and the measured overall relative combined standard uncertainty is better than 1 part in 10^{10}.

1. Introduction

An on-site comparison of Josephson Array Voltage Standards (JAVSs) at the 10 V level was carried out at PTB during the period from April 16 to April 27, 2007. The Josephson array voltage standard of PEL was used as travelling standard. The direct comparison was carried out following comparisons recently introduced by BIPM. The PTB Josephson system provided a stable and isolated reference voltage allowing routine measurement techniques to be used by the PEL Josephson system. For both systems, the accurate frequency reference of PTB (10 MHz) was used. When operated at PEL in Zagreb, a GPS disciplined oscillator for the 10 MHz Reference Frequency is used. The other difference was the helium dewar, which was not transported from Zagreb; PTB made one of its dewars available for the PEL voltage standard during the comparison.

2. Comparison equipment

2.1. The PTB JAVS

The JAVS of PTB was different to those used in previous comparisons. The idea was to follow BIPM comparisons of type "option B after the BIPM questionnaire in 2004" where PTB supplied a quantized voltage which was measured by PEL. As the conventional PTB JAVS is based on a 10-V SIS Josephson array and not floating from ground, a JAVS with a new programmable 10-V SINIS was used [1].

The 10-V SINIS array was operated using a Gunn Oscillator together with a PLL based on an EIP 578B counter. An isolated bias source supplied the bias current to the array. Polarity was reversed manually. 10-MHz π-filters and inline inductors, forming LC-filters with a cut-off frequency around 10 kHz, increase the resistance on the voltage leads to about 3 Ω. The validation of the PTB system is described in Appendix A.

2.2. The PEL JAVS

The PEL Josephson Array Voltage Standard is a commercial product of Supracon (Jena, Germany) called supraVOLTcontrol. It has the serial number 003 and is the first such system installed in a European country. It is a complete 3-channel, microprocessor-controlled Josephson voltage standard (JVS) and allows calibration of secondary voltage standards and external voltmeters. A more detailed description of the PEL Josephson system is given in Appendix B.

The main elements of this system are:

a) Cryoprobe with the 10 V Josephson Array Chip with 19700 SIS Josephson junctions and operating frequency around 75 GHz.
b) JVS Electronics Unit which controls the whole system.
c) Microwave Electronics – includes 75 GHz Gunn oscillator, isolator, directional coupler, remote sensor (mixer), and voltage controlled attenuator.
d) EIP Source Locking Microwave Counter 578B.
e) Keithley nanovoltmeter 2182 as Null Detector.
f) 3-channel Polarity Reversal Switch with special low thermal voltage cables.
g) Host Computer with IEEE interface.

The system has different modes of operation, predefined within the software. For this comparison two main modes were used: the first one was calibration of the secondary standard (section B1.1 of Appendix B, abbreviated in following text as **mode "M1"**), which was a part of software version v1. The second mode, which was added during the time of this comparison (software version v2), was programmed for a direct comparison of JAVS systems (section B1.2 of Appendix B) and is abbreviated in the following text as **mode "M2"**.

The series resistance of the measurement leads was 2.4 Ω, and the leakage resistance between the measurement leads was >130 GΩ. It should be noted that this value does not take into account the leakage due to the DVM and other elements, which was measured as explained in section 5.1.

3. Comparison procedure

Installation of the PEL JAVS system at PTB was done on the very first day, April 16, 2007, while the measurements of the series resistance of measurement leads, leakage resistance between the measurement leads and influence of the leakage resistance of the system were done on April 17–20, 2007. The repeatability of the gain correction factor of the nanovoltmeter used and the stability of the JAVS steps were also measured between April 17-20, 2007.

The comparison was divided into two parts. In the first one (April 18–20, 2007) the PEL system used mode M1 to measure the voltage of the PTB array as if it were a Zener reference; in the second part (April 24, 2007), mode M2 was used. During these measurements, the PEL array was disconnected from its bias source. The two arrays were connected in series opposition and left floating from ground.

4. Description of measurements

The PEL system has three possible (identical) channels. For the majority of the present comparison, channel A was used, and other channels are used only once (as stated in section 5). The two modes used for the PEL system (M1 and M2 as explained above) operate under complete software control. To ensure that the nanovoltmeter used operated in the best possible way, the function of the system *Calibration nanovoltmeter* (section B2.2 in Appendix B) was used at the beginning of each working day, and the corresponding gain correction factor (just for the 10-V range) was stored into the memory.

4.1. Set-up of the reference Josephson voltage by the PTB system

The reference voltage, generated by the PTB system is calculated by the known equation:

$$U_{\mathrm{PTB}}\,/\mathrm{V} = N_{\mathrm{PTB}} \times f_{\mathrm{PTB}} \,/\, K_{\mathrm{J\text{-}90}}\,,$$

where $N_{\mathrm{PTB}} = 69631$, $K_{\mathrm{J\text{-}90}} = 483\ 597.9$ GHz/V, and the frequency is set for a particular day, as reflected in Table 1.

4.2. Set-up of the frequency for the PEL system

During the measurements on April 19, 2007, the best possible frequency for the PEL system was investigated. This was done by applying the function of the system *Arbitrary Voltage* by which the system generates selected output voltage within the limits of either ±5 mV or ±2 µV. It was found that, for a chosen voltage of approximately 10 V and particular circumstances:

a) it was not possible to find stable steps for the frequencies 74.45 GHz, 74.46 GHz, 74.60 GHz and 74.77 GHz;
b) steps stable for between 5 s and 30 s with the ±5 mV limit selected and from 20 s to more than 90 s with the ±2 µV limit were found at 74.55 GHz, 74.70 GHz, 74.84 GHz, 74.85 GHz and 74.90 GHz with microwave powers between 7 mW and 16 mW.

Based on these results, the chosen set-up frequencies for the PEL system were 74.85 GHz and 74.90 GHz.

4.3. Mode M1

In this mode (section B1.1 of Appendix B), the PEL system measures the reference Josephson voltage generated by the PTB system as in the calibration of the Zener reference, as follows:

1. The system operates on the predefined frequency and measures the voltage difference within ±235 μV.
2. One measurement point consists of twenty readings for the voltage difference, ten readings at each polarity of the JJ array chip.
3. This procedure is repeated three to eight times for one polarity of the reference Josephson voltage (one sequence) during approximately 5 min.
4. The result taken into the analysis of JAVS comparison is the mean value of two sequences (+ and – polarity from the reference Josephson voltage).

4.4. Mode M2

In this mode (section B1.2 of Appendix B), the PEL system measures the reference Josephson voltage generated by the PTB system as follows:

1. The PEL system slightly changes the predefined frequency to measure a voltage difference no larger than ±1 μV.
2. One measurement point consists of plus (+) and minus (–) bias of Josephson voltages of both systems. For each polarity, 20 readings of the voltage difference are taken (one sequence).
3. Such sequence can be repeated up to 8 times.
4. The result taken into the analysis of JAVSs comparison is the mean value of all sequences (the time needed for 5 repetitions is approximately 10 min).

5. Results and uncertainties

5.1. The leakage resistance of the PEL JAVS

The leakage resistance between the measurement leads was >130 GΩ, but the leakage resistance of the whole system, which is important information for the uncertainty analysis, was determined by inserting the additional resistance R_P between the two JAVS systems (Fig. 1), or between the PEL system and a Zener reference. Measurements were performed with $R_P = 100\ \Omega$, $R_P = 1\ \text{k}\Omega$, $R_P = 2\ \text{k}\Omega$ and $R_P = 4\ \text{k}\Omega$. It was found that the leakage resistance of the PEL system is:

$$R_{\text{leak}} \approx (90 \pm 4)\ \text{G}\Omega$$

for three measurement series, with the Zener reference or in the direct comparison modes M1 or M2, respectively. As an example, the linear fit for the leakage resistance in mode M2 is shown in Fig. 1.

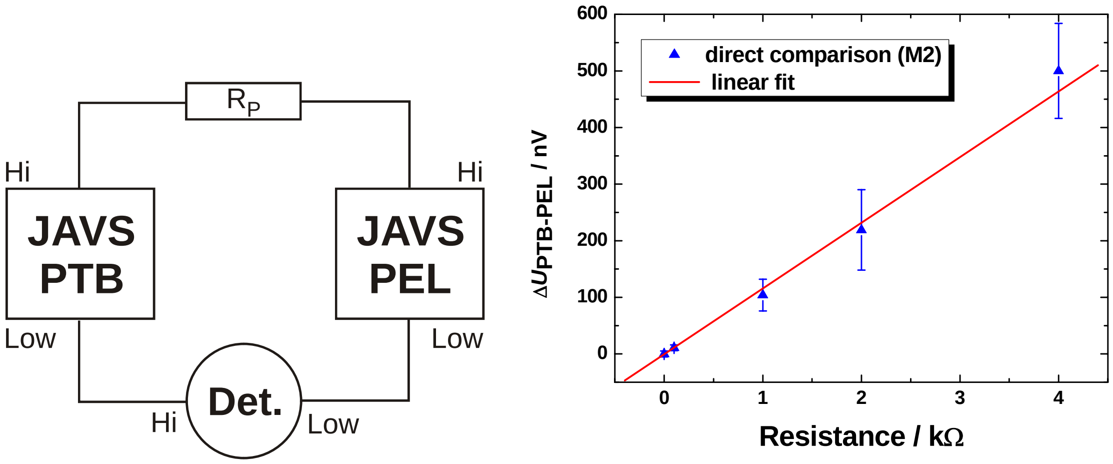

Fig. 1. Scheme of the comparison set-up (left) and linear fit of the voltage difference in dependence of resistance R_p (right).

5.2. Overview of the comparison data

The overview of the parameters of the JAVS comparison is presented in Table 1.

Table 1. Data obtained within the PTB-PEL JAVS comparison are divided into 6 series, where ordinal number of measurement identifies a particular comparison data; other parameters are described elsewhere in the text.

Series	Ordinal number	Date	PTB SINIS Voltage		PEL system		
			f_{PTB} / GHz	U_{PTB} / V	f_{PEL} / GHz	Mode	Channel
1	1 to 5	18-04-2007	70.188	10.10604187487	74.85	M1	A
2	6 to 13	19-04-2007	70.11	10.09481101965	74.85	M1	A
3	14 to 29	20-04-2007	70.11	10.09481101965	74.90	M1	A
4	30 to 39	24-04-2007	70.165	10.10273021243	74.84962	M2	A
5	40 to 49	24-04-2007	70.165	10.10273021243	74.84962	M2	B
6	50 to 59	24-04-2007	70.165	10.10273021243	74.84962	M2	C

Since the comparison between the two JAVS systems was performed using two different methods, the mean value for the voltage difference is calculated for measurements for each mode. The final value is calculated as the weighted arithmetic mean of these two values; all of these parameters are shown in Table 2, while all comparison data are shown in Fig. 2. Although for series 4, 5 and 6 different channels of the PEL system were used, the results can be taken together into the analysis because in this mode the polarity reversal switch does not change state and its influence can be neglected.

Table 2. Complete measurement results of the PTB-PEL JAVS comparison, analysed for the two modes of the comparison.

Series	Mode	Ordinal number	No. of meas.	$U_{\mathrm{PTB}} - U_{\mathrm{PEL}}$ (all values are in nV)			
				Mean	Std.dev.	Std.dev.mean	Type B
1 to 3	M1	1 to 29	29	−1.36	2.18	0.40	4.08
4 to 6	M2	30 to 59	30	−0.43	1.68	0.31	0.30

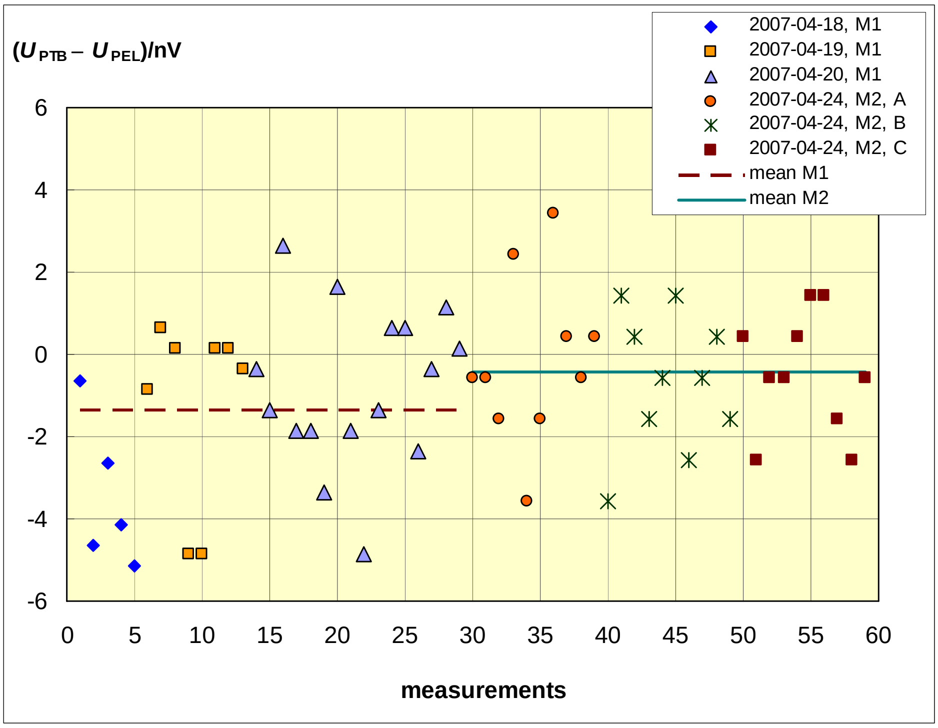

Fig. 2. All data of PEL-PTB JAVS comparison, obtained from April 18 to April 24, 2007, through 6 series of measurements; ordinal number of measurement corresponds to those in Table 1 and 2

5.3. Final result

Based on the presented data (table 2) and a weighted mean calculation on the two different types of measurement, M1 and M2, the final difference of the PEL JAVS system and the value of the PTB system is:

$$\boldsymbol{U_{\mathrm{PEL}} - U_{\mathrm{PTB}} = +0.52\ \mathrm{nV}} \qquad \textbf{or} \qquad \boldsymbol{U_{\mathrm{PEL}} / U_{\mathrm{PTB}} - 1 = +5.2 \times 10^{-11}}.$$

The detailed analysis of uncertainty contributions is presented in the Appendix C. The final statement is the relative combined uncertainty (k = 1):

$$\boldsymbol{u_{\mathrm{C}} / U_{\mathrm{PEL}} = 4.1 \times 10^{-11}}\ .$$

PTB’s SIS JAVS took part in a KC with BIPM [2]. This SIS JAVS was used to validate the SINIS JAVS which acted as a reference in this comparison. The validation is described in Appendix A.

6. Discussion and conclusion

This comparison is based on the characteristics of the PTB SINIS JAVS system to maintain a perfectly stable and reproducible 10 V output during the measurements. On the other hand, the fully automated PEL system enables predefined modes of operation, with computer-controlled JJ array and data acquisition.

The results of the comparison demonstrate the ability of the PEL JAVS system in 10-V measurements and for calibration of Zener standards. As shown in the Appendix, the direct comparison mode M2 improves very much the uncertainty for this type of measurements.

7. References

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Disclaimer

Certain commercial equipment, instruments or materials are identified in this report in order to adequately specify the environmental and experimental procedures. Such identification does not imply recommendation or endorsement by PTB or PEL, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

Appendix A

Validation of PTB's reference Josephson system

To validate the PTB JAVS comparisons between the SIS JAVS similar to that used in a previous comparison and another 10-V SINIS JAVS [2] were made.

On 20 April, during the comparison with PEL an internal comparison was made between the SINIS and SIS PTB JAVS. The systems were connected with a low thermal rotary switch from Electroswitch. The comparison showed a voltage difference of 3.1 nV with a type A uncertainty of $u_c = 0.4$ nV for two data sets of 20 measurements with both polarities. As these measurements indicated a large type B uncertainty a 1-kΩ resistance was placed between the two systems on the high voltage side to enlarge the influence of leakage currents [3]. The voltage difference increased to about 300 nV clearly indicating leakage problems for one of the systems. After the comparison with PEL this leakage problem was further investigated. The voltage difference was found to vary in the range from 20 nV to 110 nV depending on the position of the rotary switch. Removing the switch solved the problem and the voltage difference reduces to below 20 nV, demonstrating a leakage resistance of about $R_{\mathrm{leak}} \approx 1\ \mathrm{k\Omega} \times 10\ \mathrm{V} / 20\ \mathrm{nV} = 500\ \mathrm{G\Omega}$. For the direct 10-V comparison with 3-Ω leads, the uncertainty contribution due to leakage current is at maximum $10\ \mathrm{V} \times 3\ \Omega / 500\ \mathrm{G\Omega} \approx 0.06\ \mathrm{nV}$. The same measurement was made between two SINIS systems. With the 1-kΩ resistor on the high side the voltage difference was 8.1 nV ± 1.2 nV (type A) for a set of 50 measurements. This result corresponds to a leakage resistance of about $R_{\mathrm{leak}} \approx 1\ \mathrm{k\Omega} \times 10\ \mathrm{V} / 8\ \mathrm{nV} = 1.25\ \mathrm{T\Omega}$, or in other words to a type B uncertainty due to leakage of $10\ \mathrm{V} \times 3\ \Omega / 1.25\ \mathrm{T\Omega} \approx 0.02\ \mathrm{nV}$.

Finally, the SINIS PTB JAVS was validated in an automated long term measurement of the two programmable 10-V SINIS JAVS without the 1-kΩ resistor. The voltage difference for about 2500 polarity reversals in 18 hours was 0.050 nV with a type A measurement uncertainty of 0.025 nV [4].

In the final comparison with the SIS system over 32 polarity reversals, we found a deviation of -0.16 nV with a type A measurement uncertainty of 0.65 nV [4]. This type A dominates the uncertainty contributions. Therefore, the overall uncertainty for this comparison is also 0.68 nV.

Appendix B

Detailed description of the PEL Josephson system

The PEL Josephson Array Voltage Standard is a commercial product of Supracon (Jena, Germany) called **supra*VOLT*control**. Therefore, the information presented here are taken from the User Manual of Josephson Voltage Standard supraVOLTcontrol, Supracon, November 2006.

The system is a complete 3-channel microprocessor-controlled Josephson voltage standard (JVS) with a highly integrated low-Tc JJ array microwave circuit. It allows a very easy calibration of secondary voltage standards and external voltmeters. The typical relative accuracy of the system itself is better than $4 \cdot 10^{-10}$ at the level of 10 V. For calibration purposes at the same level, the noise of the secondary voltage standard limits this relative accuracy to about $1 \cdot 10^{-8}$. Additionally, it can measure the linearity and the gain factor of external voltmeters.

The elements of this system are as follows (Fig. B-1):

1. Cryoprobe with the 10 V JJ Array Chip, made by IPHT (Jena, Germany), with 19700 SIS Josephson junctions based on the standard Nb/Al trilayer technology with operating frequency around 75 GHz.
2. Liquid Helium Dewar with 50 mm flange – this is the part of the system that is not used in this PTB/PEL JAVS Comparison, because it was left in PEL in Zagreb; a similar dewar was used instead at PTB laboratory for the same purpose.
3. JVS Electronics Unit – together with the host computer, controls the whole system, including the special power supply for the microwave attenuator, and the connection to the JJ array chip.
4. Microwave Electronics – includes 75 GHz Gunn oscillator, isolator, directional coupler, remote sensor (mixer), and voltage controlled attenuator.
5. EIP Source Locking Microwave Counter 578B.
6. Keithley nanovoltmeter 2182 as Null Detector.
7. 3-channel Polarity Reversal Switch with special low thermal voltage cables.
8. Sensors for temperature, humidity and barometric pressure with cables.
9. Host Computer with IEEE interface.

The JJs are integrated in a microwave transmission line. The dc pads are connected to both fins of the finline antenna in order to supply the bias current and pick up the Josephson voltage. The JJ array chip is glued on a small printed board and its dc pads are bonded to the copper contacts of the printed board, which are soldered to the dc contacts of the chip carrier.

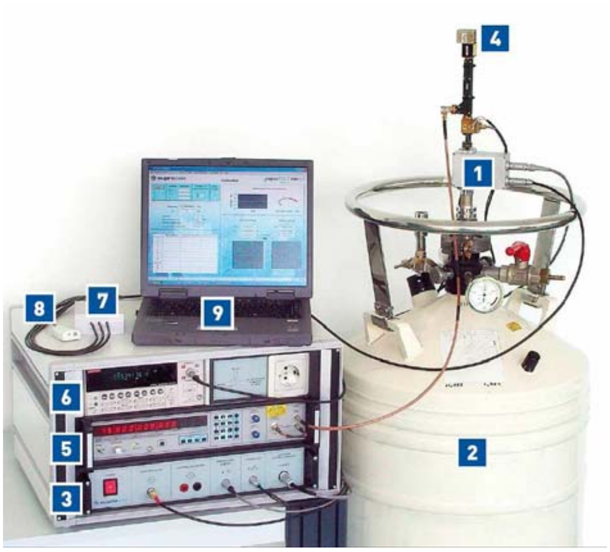

Fig. B-1. The complete 10-V JAVS system of PEL (**supra*VOLT*control**)

B1. COMPARISON MODES

B1.1. Mode M1 (Calibration of Secondary Standards)

This mode is optimised for the calibration of Zener references, whose output voltage can be connected to the JAVS through one of three identical channels (A, B and C, respectively) for any voltage level. It is also possible to disconnect the mains power supply of the DUT during the short time of the calibration measurement, if it is connected on the main socket within the system, provided that the DUT possesses a battery supply.

The JJ array is connected in series to a high resolution null detector and the secondary voltage standard which has to be calibrated. The null detector measures the difference voltage between the secondary voltage standard and the quantized voltage level of the JJ array chip. For this calibration mode, several facts are important and influence the direct comparison of JAVS systems:

1. The used microwave frequency is determined by the value saved in the set-up file of the system, and remains the same during the whole procedure.
2. Because the voltage of the Zener reference is not known, the system first measures its output voltage (on the 10 V range of the nanovoltmeter used) to calculate the approximate voltage and the closest number of steps N to be used – during this procedure one of the connections (either plus or minus) is grounded, which can cause a problem.
3. The remaining voltage difference between the output voltage of the Zener reference and the quantized JAVS voltage is measured on the 10 mV range of the nanovoltmeter (for $f = 75$ GHz, one step corresponds to approximately 155 μV). If the measured voltage difference is higher than ±235 μV the electronics adjusts to an adjacent step, where the Josephson voltage will be closer to the voltage of the secondary standard.

4. For each measurement point of the secondary standard output voltage standard, the JJ array chip is set to both polarities (Fig. B-2 and B-3). To determine each of these values (together with their standard deviation and thermal voltage), twenty readings are taken during approximately 1,5 s. A single data point is accepted if thermal voltage is less than 1 µV; this helps ensure that no voltage step jump occurred during the acquisition of that individual data point.
5. For each polarity of the JJ array chip, microwave power starts from zero milliwatts, and rise up to max. 20 mW to obtain stable steps.
6. During the calibration, the JJ array chip is completely disconnected from ground and the JVS electronics.

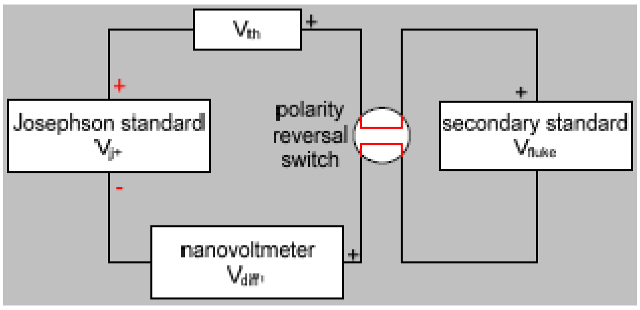

Fig. B-2. Principle of the voltage difference measurement Vdiff+ for a positive Josephson voltage Vj+ setting.

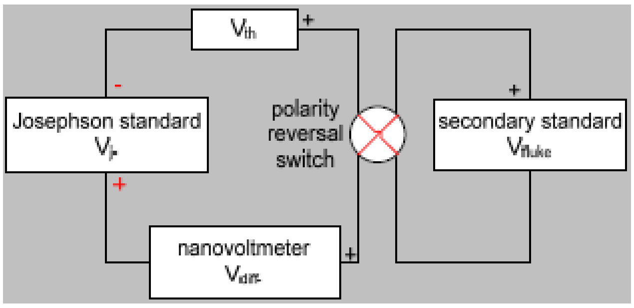

Fig. B-3. Principle of the voltage difference measurement Vdiff– for a negative Josephson voltage Vj– setting.

B1.2. Mode M2 (Direct comparison)

It is important to note the following: this new feature of the system was installed during the PTB-PEL comparison on April 23, 2007. It is programmed to enable **direct comparison of two JAVS systems** and assumes that the reference JAVS system ensures a stable Josephson voltage and this system "measures" that reference voltage. Therefore, this is based on the following:

1. In this mode, the system does not measure (approximately) the voltage relative to the grounded output.
2. Instead, it is necessary to write in the box a reference voltage value (of the reference JAVS), and the system slightly changes its predefined microwave frequency and calculates the step index *N* for which the voltage difference between two systems will be almost zero (approximately ±1 µV).

3. The polarity reversal switch remains in the same state for both polarities of the generated Josephson voltage.
4. For each polarity of the JJ array chip, microwave power starts from zero milliwatts, and rise up to max. 20 mW to obtain stable steps.
5. One measurement point consists of plus (+) and minus (−) bias of Josephson voltages of both systems, and in both cases 20 readings of the voltage difference is taken.
6. Such sequence can be repeated up to 8 times.
7. For this mode of comparison any of three channels of the system (A, B and C, respectively) can be used, but of course only one at the same time.

This measurement procedure is significantly different from mode M1 (section B1.1 of this appendix) and enables direct comparison of JAVS systems in a completely different way of operation.

B2. PERFORMANCE TESTS

B2.1. Critical current

The critical current performance test should be made with one (or two, or three) connected secondary voltage standard(s). This performance test measures the most sensitive parameter of the JJ array. The nominal value of the critical current of the installed JJ array chip is written in the control software. During the test, the current fed into the JJ array is increased step by step until the voltage drop across the JJ array jumps from zero to a certain value. The current at which this voltage jump occurs is the critical current of the JJ array. It is indicated and compared with the nominal value.

B2.2. Calibration nanovoltmeter

The gain factor of the selected voltage range for the Keithley 2182 nanovoltmeter used is calculated from 21 data points. Each data point is the average from 20 single measurements. The most important range is the 10 Volt range of the nanovoltmeter, as the measurements in this range are used to calculate the step index number N, needed for the determination of the exact Josephson voltage. The nanovoltmeter has to measure each voltage in the 10 V range with accuracy better than 50 µV in order to ensure a correct calculation of the Josephson voltage. If the 10 V range is successfully calibrated, the data are saved into the configuration file. With the newly measured gain factor, the accuracy of the nanovoltmeter is better than 10 µV in the 10 V range.

B2.3. Step flatness

It measures the flatness of the Josephson voltage step at 10 V and/or 1 V, which can be chosen. The step is flat when the Josephson voltage does not depend on the bias current through the JJ array. The step flatness performance test can be made only with a secondary voltage standard connected to the polarity reversal switch. During the test, an appropriate Josephson voltage is generated by the JVS electronics unit for different operating currents. Each data point is the average of 20 single measurements. The smallest detectable incremental resistance is limited by the noise of the connected secondary voltage standard. For the 10 V range, incremental resistances larger than 25 mΩ can be detected; for the 1 Volt range the limit is about 10 mΩ.

B2.4. Thermal voltage

This test verifies the performance of the polarity reversal switch, of the wires of each channel at room temperature, and of the wires in the cryoprobe. The indicated thermal voltage is calculated from the voltage difference between the two polarities. The thermal voltage difference should be lower than 50 nV, usually it is about 5 nV, if the wires, the short, and the relays are working properly.

Appendix C

Uncertainty analysis

The basic equation to calculate the measurement result is:

$U = N \times f / K_{\text{J-90}}\,(1 + E_{\text{leak}}) + V_{\text{det}} + V_{\text{off}} = N \times f / K_{\text{J-90}} + V_{\text{leak}} + V_{\text{det}} + V_{\text{off}}$

In Table C-1 an explanation is given of the parameters that determine the final measurement result U and its uncertainty.

Table C-1. Description of parameters

Parameter	Description	Remarks to uncertainty calculation
N	Step number to which the Josephson array is set. This parameter has no uncertainty.	0
$K_{\text{J-90}}$	The Josephson constant equal to 483597.9 GHz/V. The parameter has no uncertainty for this comparison, but it has a relative uncertainty of 4×10^{-7} with respect to the SI.	0
f	The frequency of the applied microwave.	u_f a), 1)
E_{leak} or V_{leak}	The leakage error (E_{leak}) and the corresponding voltage drop (V_{leak}) due to the finite resistance of the leads and the leakage resistance between the leads and between the leads and the ground.	u_{leak} 2)
V_{det}	The voltage difference between the Josephson arrays, which is measured with the null detector. The uncertainty of this voltage is subject to gain calibration and linearity errors, and therefore depends on the measured typical value.	u_{det} 3)
V_{off}	Residual offset voltages due to thermal EMFs in the leads and in the scanner/switch or rectification by electromagnetic interference.	u_{off} b), 4)

General remarks

a) In this comparison, only the frequency uncertainty of the transducers, the JVSs, are part of the uncertainty budget as both systems are always connected to the same time base. For comparing the value of U of NMI voltage standards, the 10 MHz time base uncertainties have to be included. In most direct Josephson comparisons these uncertainties were considered as negligible, compared with the others.

b) Uncompensated thermal EMFs usually do not appear in direct JVS comparisons as they are cancelled by reversing the bias currents of each array in both systems (the uncertainty due to their short term instabilities is already included in the type-A uncertainty of the measurements). For calibrations of electronic standards (*e.g.* Zener references), this component has to be investigated, because an intrinsic reversal of the standard is not possible.

Further Explanations:

1) The uncertainty u_f is due to a possible deviation between measured and real frequency. During this comparison, both systems use the same 10 MHz reference. A separate experiment was performed to compare the differences in readings of the EIP frequency counters. It was found out that the value of this uncertainty contribution is $u_f = 1$ Hz. The part of the frequency stability is already contained in the type A uncertainty.

2) The uncertainty u_{leak} is difficult to estimate as the path for leakage current is unknown. A common estimate for the relative uncertainty is the ratio between resistance of the measurement leads (including the filters) to the resistance between these wires and ground (including the detector): $u_{\text{leak}} = U \times (R_{\text{leads}} / R_{\text{leak}})$. For the PEL system, with $R_{\text{leads}} = 2.4\ \Omega$ and $R_{\text{leak}} = 90\ \text{G}\Omega$, it follows that $u_{\text{leak}} = 0.27$ nV.

3) The uncertainty u_{det} depends on the voltage value that is typically measured with the null detector. For mode M1, the voltage difference is measured within ±235 µV, but in mode M2 within ±1 µV. The measured gain of the nanovoltmeter used, on its 10 mV range is approximately 1.00003. As a result, the influence of this contribution can be approximated with $u_{\text{det}} = 4.07$ nV for mode M1 and $u_{\text{det}} = 0.02$ nV for mode M2, and is not part of the standard uncertainty of measurement.

4) The uncertainty u_{off} is due to uncompensated thermal EMFs and/or effects of EMI. In direct Josephson comparisons, thermal EMFs are usually cancelled out, as all parts of the measuring circuit are reversed. Any fluctuation in the thermal EMFs is already included in the standard uncertainty of the measurement.

Uncertainty budget of PTB, Germany

Output resistance of the PJVS: **3 Ω**

Leakage resistance of the PJVS: **1.25 TΩ**

Output voltage of the PJVS: **U_{PTB} = 10.102 730 212 43 V** **($U_{\text{PTB}} = N_{\text{PTB}} \times f_{\text{PTB}} / K_{\text{J-90}}$)**

Quantity	Estimate	Uncertainty	Type	Probability distribution	Standard uncertainty	Sensitivity coefficient	Uncertainty Contribution	Degree of freedom
Frequency	70.165 GHz	1 Hz	B	normal	1 Hz	142.7 pV/Hz	0.14 nV	300
V_{leak}	0 nV *	0.02 nV	B	normal	0.02 nV	1	0.02 nV	∝
V_{off}	0 nV	0.0 nV ***	B	normal	0.0 nV	1	0.0 nV	∝
U_{LAB}	10.102 730 212 43 V						0.14 nV	$\nu_{\text{eff}} = 300$

* no correction to the measured mean voltage is applied, as the path for leakage is unknown.

*** zero contribution due to polarity reversing by the two Josephson arrays without the use of a different switch positions. All stable thermal EMFs are cancelled and unstable thermal EMFs give a type A contribution in the standard deviation.

Uncertainty budget of PEL, Croatia (mode M1)

Output voltage of the PJVS: $\mathbf{U_{PTB} = 10.094\ 811\ 019\ 65\ V}$ $(U_{PTB} = N_{PTB} \times f_{PTB} / K_{J\text{-}90})$

Quantity	Estimate	Uncertainty	Type	Probability distribution	Standard uncertainty	Sensitivity coefficient	Uncertainty Contribution	Degree of freedom
Measured mean voltage	10.1 V	0.40 nV	A	normal	0.40 nV	1	0.40 nV	28
Frequency	75 GHz	1 Hz	B	normal	1 Hz	134.9 pV/Hz	0.13 nV	300
V_{leak}	0 nV *	0.27 nV	B	normal	0.27 nV	1	0.27 nV	11
V_{det}	235 μV (M1) **	7.05 nV	B	rectangular	4.07 nV	1	4.07 nV	100
V_{off}	0 nV	0.0 nV ***	B	normal	0.0 nV	1	0.0 nV	∞
U_{LAB}	10.094 811 021 01 V						4.10 nV	$\nu_{eff} = 103$
Voltage difference $U_{PTB} - U_{PEL}$	−1.36 nV						4.10 nV	$\nu_{eff} = 103$

The final result of the calibration in mode M1 is:

$\mathbf{U_{PTB} / U_{PEL} - 1 = -1.36 \times 10^{-10}}$; $\mathbf{u_C / U_{PEL} = 4.1 \times 10^{-10}}$

where u_C is the combined uncertainty (k = 1).

* no correction to the measured mean voltage is applied, as the path for leakage is unknown.

** typical voltage at the null detector, that is already part of the mean measured voltage. Therefore, this voltage must not be added to the mean measured voltage to get U_{PJVS}.

*** zero contribution due to polarity reversing by the two Josephson arrays without the use of different switch positions. All stable thermal EMFs are cancelled and unstable thermal EMFs give a type A contribution in the standard deviation.

Values of the laboratory

Frequency:	75 GHz
Series resistance of leads/filters:	2.4 Ω
Leakage resistance:	90 GΩ
Typical voltage at null detector	235 μV
Null detector and settings:	Keithley 2182, 10 mV range, no filters
Measurement sequence	+/– bias of PEL system, 3 to 8 repetitions, each repetition 20 readings of null detector; the same procedure for +/– bias of PTB system
Typical time for sequence	2 × 5 minutes

Uncertainty budget of PEL, Croatia (mode M2)

Output voltage of the PJVS: $\mathbf{U_{PTB} = 10.102\,730\,212\,43\ V}$ $(U_{PTB} = N_{PTB} \times f_{PTB} / K_{J\text{-}90})$

Quantity	Estimate	Uncertainty	Type	Probability distribution	Standard uncertainty	Sensitivity coefficient	Uncertainty Contribution	Degree of freedom
Measured mean voltage	10.1 V	0.31 nV	A	normal	0.31 nV	1	0.31 nV	29
Frequency	75 GHz	1 Hz	B	normal	1 Hz	134.9 pV/Hz	0.13 nV	300
V_{leak}	0 nV *	0.27 nV	B	normal	0.27 nV	1	0.27 nV	11
V_{det}	1 µV (M2) **	0.03 nV	B	rectangular	0.02 nV	1	0.02 nV	100
V_{off}	0 nV	0.0 nV ***	B	normal	0.0 nV	1	0.0 nV	∝
U_{LAB}	10.102 730 212 86 V						0.43 nV	$\nu_{eff} = 43$
Voltage difference $U_{PTB} - U_{PEL}$	–0.43 nV						0.43 nV	$\nu_{eff} = 43$

The final result of the calibration in mode M2 is:

$\mathbf{U_{PTB} / U_{PEL} - 1 = -4.3 \times 10^{-11}}$; $\mathbf{u_C / U_{PEL} = 4.3 \times 10^{-11}}$

where u_C is the combined uncertainty (k = 1).

* no correction to the measured mean voltage is applied, as the path for the leakage is unknown.

** typical voltage at the null detector, that is already part of the mean measured voltage. Therefore, this voltage must not be added to the mean measured voltage to get U_{PJVS}.

*** zero contribution due to polarity reversing by the two Josephson arrays without the use of a different switch positions. All stable thermal EMFs are cancelled and unstable thermal EMFs give a type A contribution in the standard deviation.

Values of the laboratory

Frequency:	75 GHz
Series resistance of leads/filters:	2.4 Ω
Leakage resistance:	90 GΩ
Typical voltage at null detector	1 µV
Null detector and settings:	Keithley 2182, 10 mV range, no filters
Measurement sequence	+/– bias of both systems, 5 repetitions, each repetition 20 readings of null detector
Typical time for sequence	10 minutes